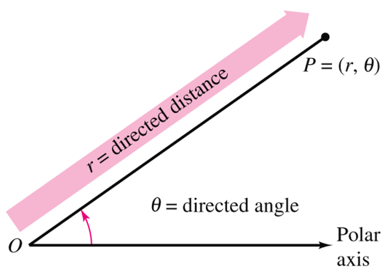
Math 155, ***Lecture Notes- Bonds*** **Name____________**

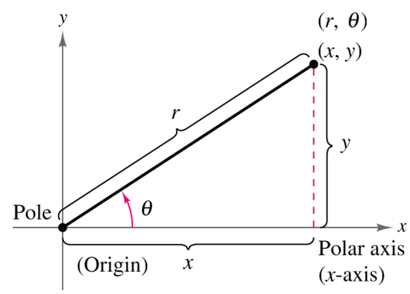
Section 10.4 *Polar Coordinates and Polar Graphs*

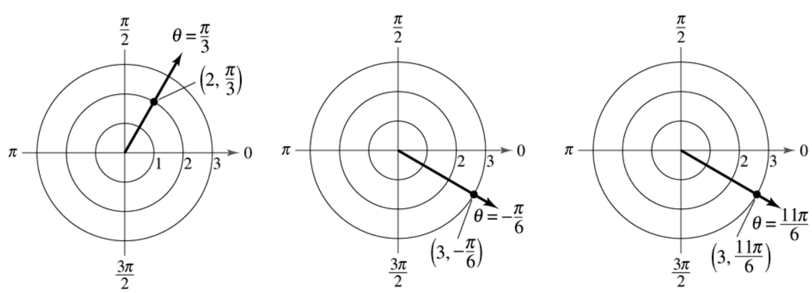
Now that we can graph parametric equations, we can study a special parametric system called the **polar coordinate system**. To form the polar coordinate system in the plane, we fix a point O, called the pole, or origin, and construct from O an initial ray called the **polar axis**, as shown below. Then each point P in the plane can be assigned polar coordinates (r,θ), as follows.

$r =$ *directed distance* from O to P

$\theta =$ *directed angle*, counterclockwise from polar axis to segment $\overline{OP}$

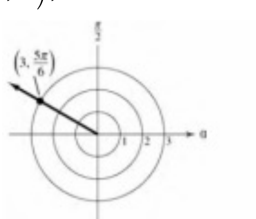






(a) (b) (c)

With rectangular coordinates, each point (x,y) has a unique representation. This is not true with polar coordinates. For instance: $(r,\theta) = (r,\theta + 2n\pi)$, or $(r,\theta) = (-r,\theta + [2n+1]\pi)$, where n is any integer. In fact, the pole can be represented by $(0,\theta)$, where θ is any angle.

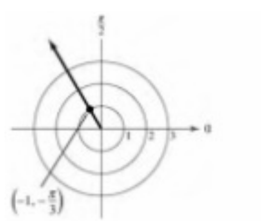


Three additional representations:

$\left(3, \frac{5\pi}{6} - 2\pi\right) = \left(3, -\frac{7\pi}{6}\right)$

$\left(-3, \frac{5\pi}{6} + \pi\right) = \left(-3, \frac{11\pi}{6}\right)$

$\left(-3, \frac{5\pi}{6} - \pi\right) = \left(-3, -\frac{\pi}{6}\right)$

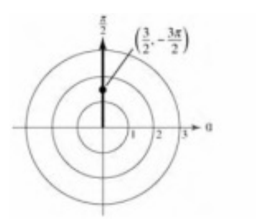


Three additional representations:

$\left(-1, -\frac{\pi}{3} + 2\pi\right) = \left(-1, \frac{5\pi}{3}\right)$

$\left(1, -\frac{\pi}{3} + \pi\right) = \left(1, \frac{2\pi}{3}\right)$

$\left(1, -\frac{\pi}{3} - \pi\right) = \left(1, -\frac{4\pi}{3}\right)$



Three additional representations:

$\left(\frac{3}{2}, -\frac{3\pi}{2} + 2\pi\right) = \left(\frac{3}{2}, \frac{\pi}{2}\right)$

$\left(-\frac{3}{2}, \frac{-3\pi}{2} + 3\pi\right) = \left(-\frac{3}{2}, \frac{3\pi}{2}\right)$

$\left(-\frac{3}{2}, \frac{-3\pi}{2} + \pi\right) = \left(-\frac{3}{2}, -\frac{\pi}{2}\right)$

THEOREM 10.10 Coordinate Conversion

The polar coordinates (r, θ) of a point are related to the rectangular coordinates (x, y) of the point as follows.

1. $x = r\cos\theta$ **2.** $\tan\theta = \dfrac{y}{x}$

$y = r\sin\theta$ $r^2 = x^2 + y^2$

Ex. 1: Convert $\left(\dfrac{5}{2}, \dfrac{4}{3}\right)$ in rectangular coordinates to polar coordinates.

Ex. 2: Convert the equation $r = \dfrac{2}{\sin(\theta)}$ from polar coordinates to rectangular coordinates and sketch its graph.

Ex. 3: Convert the equation $\left(x^2 + y^2\right)^2 - 9\left(x^2 - y^2\right) = 0$ from rectangular coordinates to polar coordinates and sketch its graph.

Ex. 4: Draw the graph of $r = 2 + 3\sin(\theta)$ and complete the table.

θ	$\sin(\theta)$	$3\sin(\theta)$	$2+3\sin(\theta)$

Ex. 5: Draw the graph of $r = 2 - \cos(\theta)$ and complete the table.

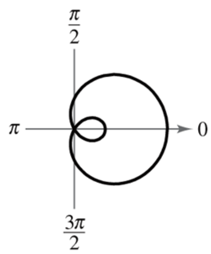
θ	$\cos(\theta)$	$-\cos(\theta)$	$2-\cos(\theta)$

Limaçons

$r = a \pm b \cos \theta$

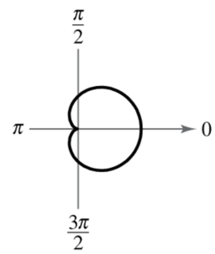
$r = a \pm b \sin \theta$

$(a > 0, b > 0)$



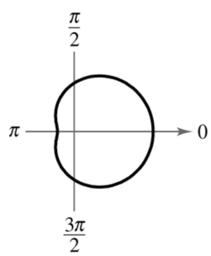
$\frac{a}{b} < 1$

Limaçon with inner loop



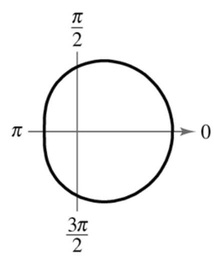
$\frac{a}{b} = 1$

Cardioid (heart-shaped)



$1 < \frac{a}{b} < 2$

Dimpled limaçon



$\frac{a}{b} \geq 2$

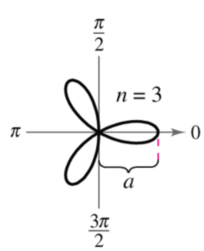
Convex limaçon

Rose Curves

n petals if n is odd

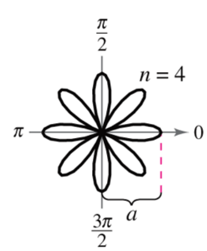
$2n$ petals if n is even

$(n \geq 2)$



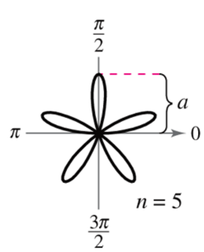
$r = a \cos n\theta$

Rose curve



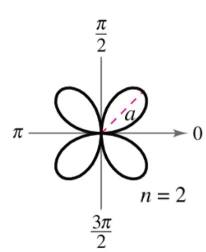
$r = a \cos n\theta$

Rose curve



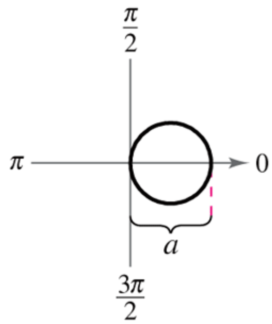
$r = a \sin n\theta$

Rose curve

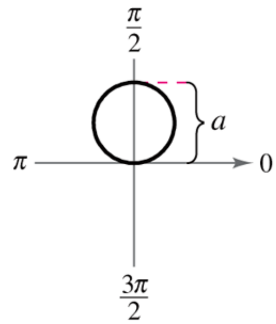


$r = a \sin n\theta$

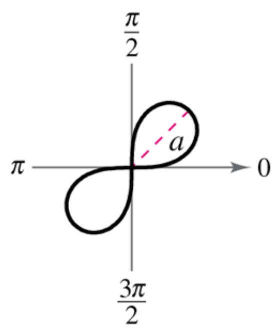
Rose curve



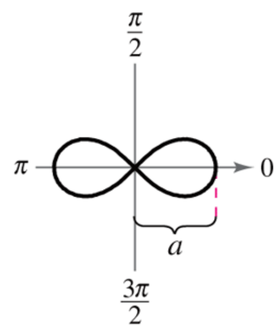
$r = a \cos \theta$
Circle



$r = a \sin \theta$
Circle



$r^2 = a^2 \sin 2\theta$
Lemniscate



$r^2 = a^2 \cos 2\theta$
Lemniscate

Circles and Lemniscates

We can use the parametric form of $\frac{dy}{dx}$ to establish the polar form of the derivative. This will allow us to find the slopes of tangent line at particular points on polar curves.

THEOREM 10.11 Slope in Polar Form

If f is a differentiable function of θ, then the *slope* of the tangent line to the graph of $r = f(\theta)$ at the point (r, θ) is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

provided that $dx/d\theta \neq 0$ at (r, θ). (See Figure 10.45.)

Ex. 6: Find $\frac{dy}{dx}$ and the slopes of the lines at the given points for the given curve:

$r = 2\left[1 - \sin(\theta)\right]$ at **A.** $(2,0)$, **B.** $\left(3, \frac{7\pi}{6}\right)$, and **C.** $\left(4, \frac{3\pi}{2}\right)$.

More Ex. 6:

Ex. 7: **(a)** Use a graphing utility to draw the graph of $r = 3 - 2\cos(\theta)$ and complete the table. **(b)** Use a graphing utility to draw the tangent line at $\theta = 0$.

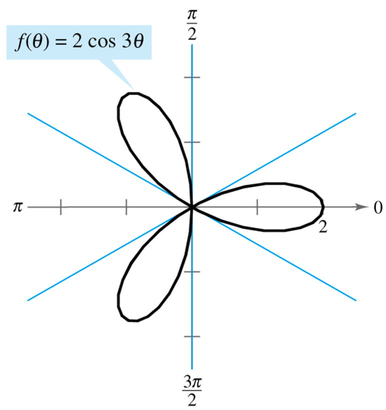
(c) Find $\left.\dfrac{dy}{dx}\right|_{\theta=0}$.

θ	$\cos(\theta)$	$-2\cos(\theta)$	$3-2\cos(\theta)$

More Ex. 7:

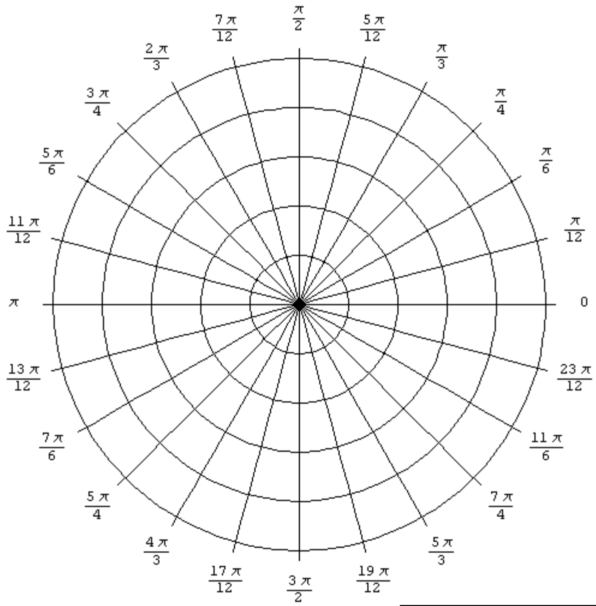
THEOREM 10.12 Tangent Lines at the Pole

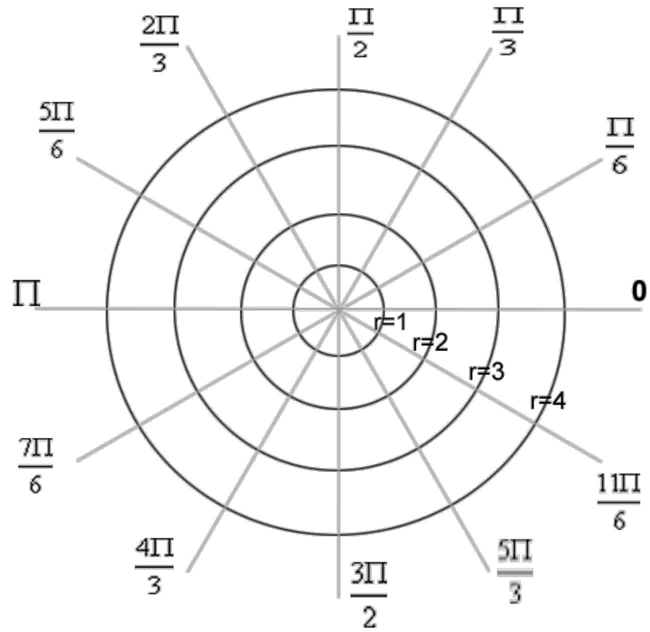
If $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, then the line $\theta = \alpha$ is tangent at the pole to the graph of $r = f(\theta)$.



Ex. 8: Sketch the graph of $r = 3\cos(2\theta)$ and find the tangents at the pole.

Polar System Grids:





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